

SUCCESS ACADEMY EDUCATION INSTITUTE

Success Academy Middle School Math Routines

Overview of Math Routines

Math Routines are repeated activities that support middle school students in becoming confident users of mathematics, powerful quantitative thinkers, and productive problem solvers. Math Routines encourage students to think deeply about problems and reason about the information in them, construct viable arguments based in mathematical reasoning, justify their own understanding in mathematical ideas, and persevere through challenging problems by using familiar mathematical ideas to make sense of new and unfamiliar situations.

Math Routines are mini-lessons that launch instruction 2-3 times per week and take anywhere from 10-30 minutes to complete based on student needs and progress. The Math Routines included can be applied in any middle school grade unless specified otherwise.

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Number Strings

Number Strings are mini-lessons that enhance scholars' numeracy, fluency, and confidence with math. When executed at a high level, Number Strings encourage scholars to visualize numbers and operations and to choose strategies that make problems friendlier. Number Strings look deceptively simple, but they require careful planning and flexible execution.

Planning for Number Strings:

Step 1: Choose the one goal you will drive towards in this mini-lesson.

- Solve all problems to understand the relationships between them.
- Identify the relationships in this string that will help scholars solve other problems in the future.
- Use trends in scholar work and misconceptions to determine **one high leverage takeaway** that you will use this string to drive towards. What do you want scholars to say when you ask them for this takeaway at the end of the string?

- What will you look for in scholar work to know if they are applying this takeaway to future problems?

Step 2: Determine how you will represent scholar thinking visually to drive toward your goal.

- What model or models will you use to represent scholars' thinking in order to highlight key relationships?
- Practice drawing models for each problem so that you have a sample that shows what you want your chart paper to look like when you are done.

Step 3: Plan your launch and context.

- What story or context will naturally encourage scholars to imagine the model you want scholars to use?
- How will you tell this story succinctly? How will you set scholars up to use the story and model to make sense of the numbers in the string?
- **Notes on contexts for strings:**
 - Contexts for strings must describe situations that scholars can **imagine** and **envision**. They should be fairly simple.
 - Contexts support scholars in exploring and developing new concepts, models, and strategies. Contexts help convince scholars that the relationships they see are valid.
 - For example, a scholar can envision a box with 8 rows of 14 chocolates each as two boxes with 4 rows of 14 chocolates each put together. This supports scholars with seeing why 8×14 is equivalent to $(4 \times 14) + (4 \times 14)$.

Step 4: Plan your pacing.

- Identify the questions in the string where scholars will meet your goal. Plan to spend the most time on these questions.
- Plan to move quickly through “helper questions” that you expect will be easy for scholars.
- Identify repetitive groups of problems so that you can skip these if you are pressed for time.

Best Practices for Implementing Number Strings:

Implementing successful number strings requires flexibility. Well-planned strings fall flat when teachers inflexibly rely on pre-scripted plans without responding in the moment to what scholars actually say. Evaluate whether you are making progress toward your goal by listening authentically to scholars and asking questions that focus their thinking about the concepts, models, or strategies you are working to develop.

Ask; do not tell.

Asking questions requires scholars to do most of the talking, and thus most of the thinking.

- Ask scholars how they are visualizing the problems, and represent their thinking for the class as they share. Be strategic and ask follow up questions to clarify how and why the visual models represent the problems.
- Ask scholars to confirm or reject answers based on each other's justifications. Do not confirm answers as the teacher.
- Ask if anyone else solved the problem using a different strategy. Have scholars re-state each other's strategies in their own words.

Focus on the thinking.

Number Strings include problems that all scholars could solve with pencil and paper, so getting to the right answer is a very small part of this routine. Agreeing on an answer must be the beginning of the conversation, not the end.

- Ask how scholars came to their answers and how to show their strategies visually.
- Value multiple approaches. When comparing multiple strategies, listen during turn and talks and strategically call on scholars who's thinking will help you drive toward your goal.
- Celebrate skeptical scholars! Press others to convince skeptics by explaining their thinking in new ways.
- Play the skeptic yourself. If you do not hear or see misconceptions that you anticipated, introduce these to the conversation as your own strategies and ask scholars to evaluate them.

Explicitly question scholars about relationships between problems.

In isolation, the problems in most strings are neither challenging nor interesting. The point is for scholars to compare different problems and strategically manipulate challenging problems to make them friendlier.

- Ask scholars how to use a previous problem or model to solve the next one.
- Ask scholars what pattern they notice in this string.
- Ask scholars to predict what problem might come next.
- Say, "You're never going to see this exact string again. What are you taking away to help you with future math problems?"

Move quickly to keep engagement up.

Not all questions will take the same amount of time. Plan for which questions are worth spending time on. Strategically ask for multiple answers or strategies on challenging or high-leverage problems.

Execution of Number Strings

In a Number String, your job is to put up the problems one at a time, then ask scholars to share how they solved, and represent their thinking on a visual model for the whole class to see. As scholars share their thinking, press them to tell you exactly what they are envisioning on the visual model so that you can represent their thinking accurately. Keep the problems and visual models displayed as you progress through the string so that scholars can make connections to previous problems and models.

Multiplication Number Strings

As you work through this string, represent scholar strategies with arrays and equations. When multiplying a single-digit number and a multi-digit number, it's often helpful to multiply the single digit by the value of each digit in the larger number, and then add the products. For instance,

129×8 can be solved by 100×8 , 20×8 , 9×8 , and then adding the products. The goal is for scholars to understand why this works and why it's helpful, which will set them up to understand the standard multiplication algorithm.

$$100 \times 8$$

$$20 \times 8$$

$$9 \times 8$$

$$200 \times 8$$

$$4 \times 8$$

$$60 \times 8$$

$$264 \times 8$$

$$304 \times 5$$

As you work through this string, represent scholar strategies with arrays and equations. Emphasize the idea that a certain strategy can be represented using different models. (For instance, the same partial products strategy for 11×13 could be shown with an equation as $(10 \times 10) + (10 \times 3) + (1 \times 10) + (1 \times 3) = 143$, or as an array that models breaking up and multiplying the same way.) Understanding this idea will set scholars up for success in the discourse, when they must recognize similarities between the standard algorithm and other methods of multiplication.

$$10 \times 13$$

$$11 \times 13$$

$$9 \times 13$$

$$20 \times 13$$

$$21 \times 13$$

$$19 \times 13$$

$$101 \times 13$$

Division Number Strings

Work through the string below, representing scholar strategies on the array. Focus on partial quotients strategies, and why they work. This will set scholars up to use partial quotients on larger numbers during the Explore, and to think about this strategy deeply in preparation for the Discourse.

$$21 \div 3$$

$$30 \div 3$$

$$51 \div 3$$

$$81 \div 3$$

$$140 \div 14$$

$$154 \div 14$$

$$280 \div 14$$

$$294 \div 14$$

Division Number Strings

Work through the string below, representing scholar strategies on the array. Allow scholars to debate the best strategy for various problems: Are partial quotients or multiplication easier? What's the best way to break the numbers down? How helpful is an array? Scholars should have mastered these strategies and models. Now they must think strategically about when to deploy each one.

$$28 \div 7$$

$$70 \div 7$$

$$98 \div 7$$

$$168 \div 7$$

$$170 \div 17$$

$$187 \div 17$$

$$340 \div 17$$

$$357 \div 17$$

Multiplication and Division Number Strings

The goal of this string is to use the relationship between multiplication and division to illuminate different interpretations of division.

In each problem in this string, Kara the kangaroo is jumping along a track. Scholars must determine the number of jumps needed to reach the target given the length of the kangaroo's jump and the total distance to the target.

$$\underline{\hspace{1cm}} \times 12 = 60$$

- Kara jumps by 12's and the target is located at 60 on the track. How many jumps does she need to take to reach the target?

$$\underline{\hspace{1cm}} \times 6 = 60$$

- What if Kara jumps by 6's? "How can we use our last picture to help us answer this?"

$$6 \times \underline{\hspace{1cm}} = 60$$

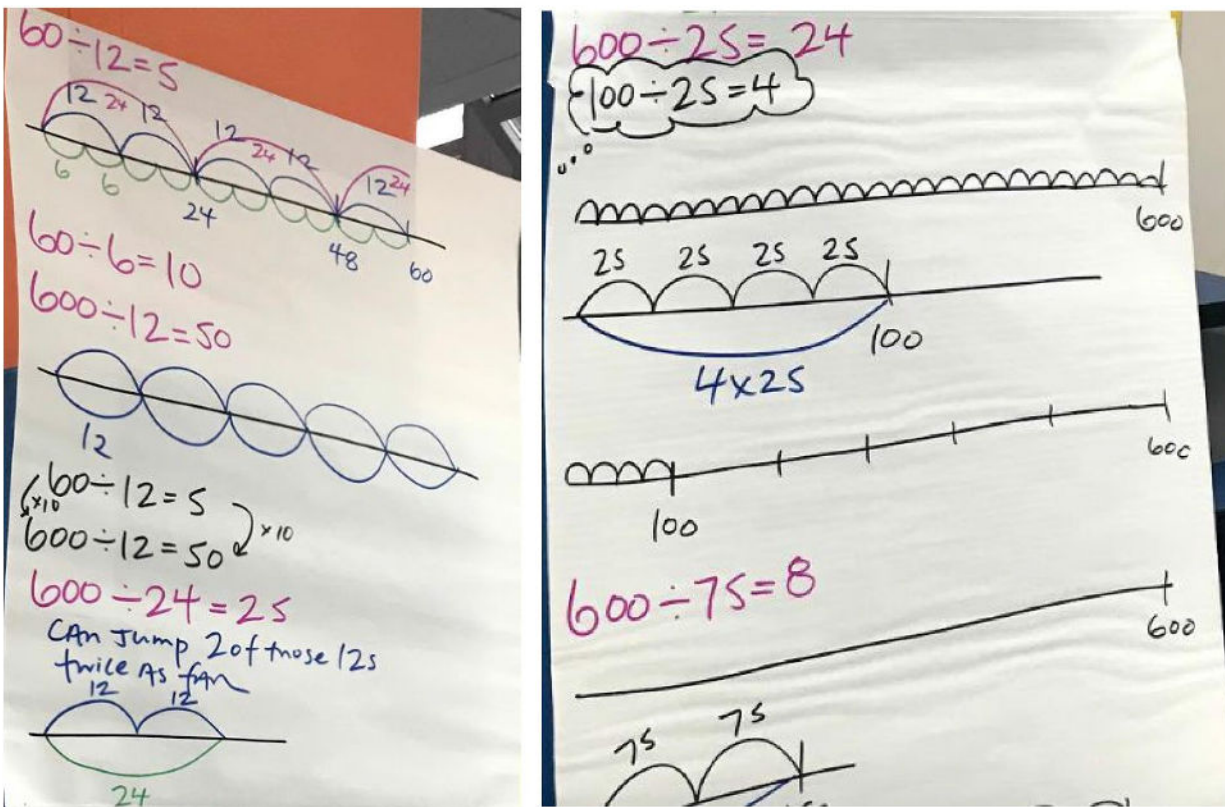
- What if I know Kara took 6 jumps to get to 60 instead? "How does this change the picture?"

$$6 \div \underline{\hspace{1cm}} = 6 \text{ – "What could this question mean in the context of our story?"}$$

$$\underline{\hspace{1cm}} \times 5 = 60$$

– If Kara jumps by 5's, how many jumps does it take to get to 60? "Which picture can you use to help you answer this question?"

$60 \div \underline{\quad} = 5$ – "What could this question represent in our story? How does the picture change?"



$\underline{\quad} \times 12 = 600$ – If Kara jumps by 12's, how many jumps does it take to get to 600?

$12 \times \underline{\quad} = 600$ – What if Kara takes 1 jumps? "Which situation is easier for you to picture? Why?"

$600 \div \underline{\quad} = 24$ – "What might this question mean in our story?"

$600 \div \underline{\quad} = 25$ – "How can we use the previous question to help us answer this one?"

$600 \div \underline{\quad} = 75$

Number String

The goal of this string is for scholars to understand that scaling the dividend and divisor by the same amount does not change the size of the quotient.

$24 \div 4$ – "How many groups of 4 are in 24? How can we draw this?"

$6 \div 1$ – “How many groups of 1 are in 6? How is our model for this question similar to or different than the last one?”

$2 \div 13$ – “How many groups of 13 are in 2? How is this model similar to or different than the last one?”

$4 \div 13$ – “Will the quotient be larger or smaller than the last question? How do you know?”

$3 \div 14$ – “How is this model similar to or different than the last one?”

$3 \div 24$ – “Will this quotient be larger or smaller than the last one?”

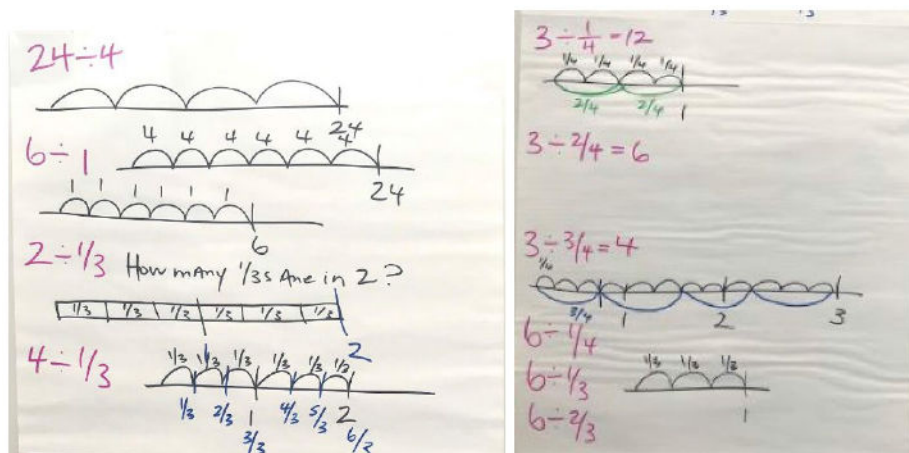
$3 \div 34$ – “Will this quotient be greater than 1 or less than 1?” and “Which previous problem helps you the most with this one?”

$6 \div 14$ – “Which diagram can you use to help you answer this question?”

$6 \div 13$ – “Which diagram can you use to help you answer this question?”

$6 \div 23$

$4 \div 25$ – “What friendlier problem would you solve first to help you answer this question?”



Count Around the Room

Adapted from [Contexts for Learning](#), [Mini-lessons for Extending Addition and Subtraction](#), “Count Around the Room” provides students with an opportunity to familiarize themselves with the names and sequence of numbers while also encouraging scholars to notice patterns in sequences and make predictions on how a pattern might continue. During Count Around the Room scholars count or skip-count by a given amount, each scholar saying one number. After scholars have gone around the room, they then discuss the patterns they noticed in the resulting number sequence.

Structure of Count Around the Room:

Count Around the Room Structure
Launch (1 minute)
Have the class sit in a circle. Tell students the start number and how they are going to count (by 1's, 2's, 5's, 10's, etc — in older grades this may also include fractions).
Activity (3 minutes)
Starting with the first person, each scholar says a number. Continue to go around the circle. Stop pointing as you go, holding scholars accountable to know when it is their turn. As scholars count, record their numbers vertically on the chart paper — recording vertically will help scholars notice patterns during the discourse. Stop at select points to ask purposeful questions and make predictions — this will help keep scholars engaged in thinking about counting. Ask scholars to consider what number comes next and how they know.
Discourse (10 minutes)
Pose the following questions for scholars to discuss. • What do you notice? • Why do you think that happens? • Will this always happen? • Do you agree or disagree? Why? Possible conjectures: • Counting patterns - the numbers 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 repeats • Place value patterns ○ For example: In Base 10, each place is worth 10 times the place to its right. Adding 10 tens, for example, is the same as adding 1 hundred.

Exit Ticket (1 minute)
Write down a few numbers and ask scholars to fill in the blank— include counting by 1s, 2s, 5s, and 10s. To vary the format of these questions: Make it a story problem or list the numbers vertically and horizontally. Ask scholars how they know and can they identify the pattern.

Top Instructional Moves for Count Around the Room:

- Focus on patterns, structure, and place value. Question scholars about how and why digits change or stay the same. Then press them to explain these patterns using what they know about place value. This mini-lesson is not a fluency exercise that teaches number sequence/number names.
- Use purposeful numbers: Choose your starting number and what you count by purposefully. Use scholar data and observations from work study to inform your choices. For example, if scholars are having trouble working with numbers that extend to the thousands place, purposely start with a number in the hundreds and count up to a four-digit number so that scholars can see that place value patterns continue even as numbers get bigger.
- Record scholar thinking: As scholars discuss noticings and patterns in counting, record their thinking, including any conjectures.
- Prepare a chart with two vertical columns. One column labeled decimals, the other labeled fractions.
 - First Count by multiples of one tenth starting with one tenth. Be sure to record what each scholar says, including a variety of decimal and fraction notation and As scholars are counting, stop and discuss:
 - What patterns do you notice?
 - How else could we say/write this number?
 - How many wholes are there in ten tenths? How do you know?
 - Is 2.4 (make sure to read as two and four tenths) the same as 24 tenths? Convince me.
 - Then, count by hundredths starting with $\frac{1}{100}$
- Record two columns on chart paper: one column indicating the fraction and another indicating the decimal.
 - After a scholar says $\frac{10}{100}$, scholars turn and talk about how that should be represented as a decimal. Discuss why it's 0.1 or 0.10 rather than 0.010. Use visual models as necessary.
 - Continue counting around the room, pausing to quickly discuss representation as necessary.
 - Record the hundredths in another two columns next to the tenths.
 - If these first two are easy and scholars are able to complete them quickly, announce you'll count by $\frac{3}{100}$, beginning with $\frac{3}{100}$.
 - Guide the discussion after the count around the room to focus on the relationship between digits in the tenths place and the hundredths place. Scholars should be able to articulate that digits in the tenths place are worth ten times as much as in the hundredths place.

Sample Count Around the Room Lessons:

- First, count by $\frac{3}{10}$, beginning with $\frac{3}{10}$
 - Record two columns on chart paper: one column indicating the fraction and another indicating the decimal.
 - After a scholar says $\frac{12}{10}$, scholars turn and talk about how that should be represented as a decimal. Discuss why it's 1.2 rather than 0.12. Use visual models as necessary.
 - Continue counting around the room, pausing to quickly discuss representation as necessary.
- If the first routine goes quickly, Count around by $\frac{11}{1000}$
- Have students predict what number they'll say before beginning to count. As scholars count, the teacher records the decimal and fraction equivalent. Pause at 0.033, 0.110, and 0.143, and have scholars help you model these amounts with the blue place value blocks. Target any misconceptions you noted in Lesson 2. This lesson is not effective unless you record numbers as you are doing the routine so scholars can associate the number with the name.

Closer To

"Closer To" helps students build their skills within rounding. When problem-solving, this skill can help scholars to estimate and determine the reasonability of their answer. This mini-lesson asks scholars to round a number to a given whole number or fraction using the number line model as a visual representation.

Structure of Closer To:

Closer To Mini-Lesson Structure
Launch (1 minute)
Introduce a class number line with two numbers filled in at the start and end point. (e.g. 100 on the left side of the line and 200 on the right side of the line)
Activity (3 minutes)

Introduce another number that falls between the first two. (e.g. 182) Ask scholars where it would go on the number line and which of the other two numbers it's closer to.

Scholars use whiteboards to plot their answers and discuss with partners. The whole class discusses which of the 2 numbers it's closer to and how they know.

Mark it on the class number line.

Repeat this with more numbers that fall between the original two numbers, always asking scholars to demonstrate their reasoning.

Discourse (10 minutes)

Discuss patterns that occur and make conjectures about rounding, place value, or related mathematical ideas.

Exit Ticket (1 minute)

Provide scholars with an exit ticket to assess the goals of the mini-lesson.

Top Instructional Moves for Closer To:

- **Choosing Numbers:** Be purposeful in your number choices— both the numbers you use as the range for the number line and the numbers you ask scholars to plot. Use scholar work, gaps in understanding, and information from the Math lesson to inform your choices and sequence. Numbers are typically provided in the lesson plan.
- **Use models rather than rhymes or rules:** Illustrate scholars' thinking using number lines to support conceptual understanding. Teaching rounding "rules" or tricks puts the focus on memorization without true understanding.
- **Valorize a variety of strategies:** Scholars may find it helpful to locate the midpoint between the two endpoints, to calculate the difference between the given number and each end point, or to use some other strategy. Acknowledge that there are a variety of ways to think about the problem, and that certain strategies may work better for certain numbers. Do not make blanket mandates for scholars to use a specific strategy every time.

Closer To Example:

- Is 0.456 closer to 0 or 1? Closer to 0.4 or 0.5?
- Is 0.381 closer to 0 or 1? Closer to 0.3 or 0.4? Closer to 0.38 or 0.39?
- Is 0.095 closer to 0 or 1? Closer to 0.09 or 0.10? Why do we generally "round up"?

Connecting Representations

Adapted from [Fostering Math Practices](#)

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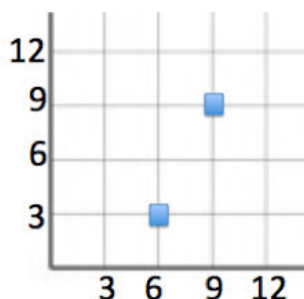
Grade 5

The purpose of this mini-lesson is to reinforce the importance of looking carefully at the scale of the axes when finding distances on the coordinate plane. Show scholars all representations below. Ask them which description and coordinates match which points on the graph. Give scholars time to think independently and discuss with a partner before sharing out. Once all scholars are convinced of the representations that match, have them write the coordinates and a description of the remaining point and plot the point with the remaining coordinates on the graph.

Key Questions:

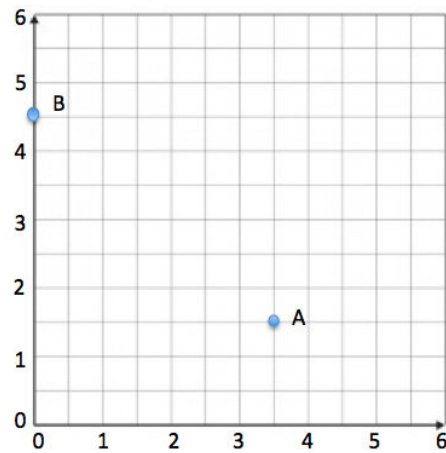
- Which description matches each point? How do you know?
- How are the coordinates of a point related to its distance from the x- and y- axes?
- What is the distance between each grid-line? How do you know?
- Why is it important to look carefully at the axes before finding distances on the coordinate plane?

Example 1



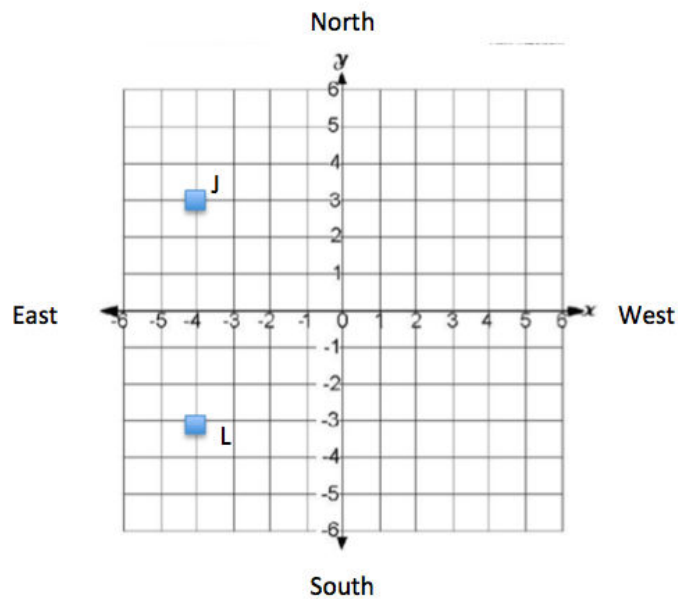
- 6 units from the y- axis and 3 units from the x- axis
 - (9, 9)
 - (0, 12)

Example 2



- A point that is zero units away from the y- axis and 4.5 units away from the x- axis
 - $(3.5, 1\frac{1}{2})$

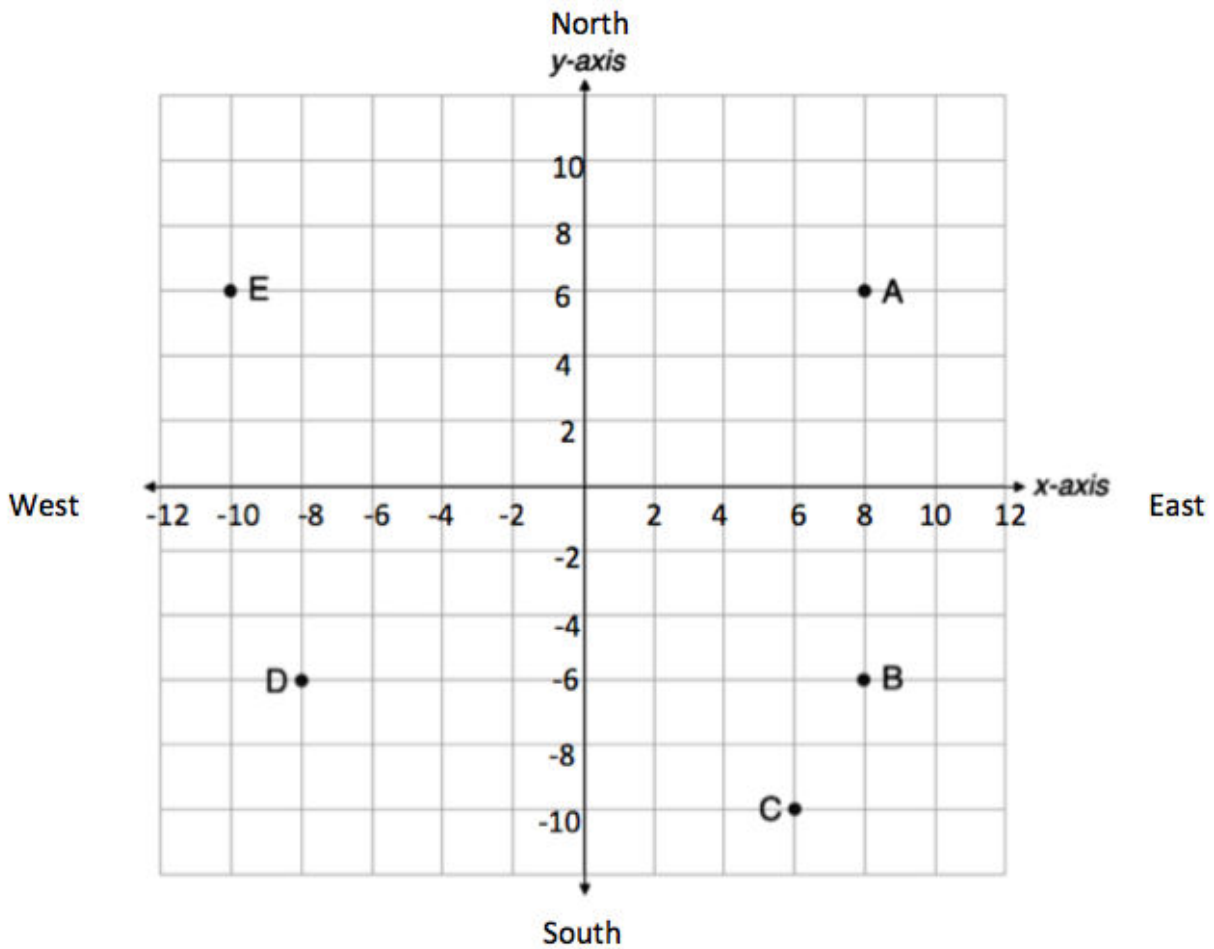
Example 3



Martine has used a coordinate plane to model a soccer field to show her teammates some plays she has planned for their next game. The points plotted above represent some of the players on Martine's team. Determine which description matches which point, and plot the point that is missing.

- 6 units west of a teammate, and 3 units north of the x- axis
 - $(-4, 3)$
- 3 units south of the x- axis and four units east of the y- axis

Example 4



- (-8, -6)
- 6 units north of the *x-axis* and 8 units west of the *y-axis*
 - (6, -10)
- 8 units east of the *y-axis* and 6 units north of the *x-axis*
 - (8, -6)
- What is the distance between point B and point A?

Grade 6

Example 1

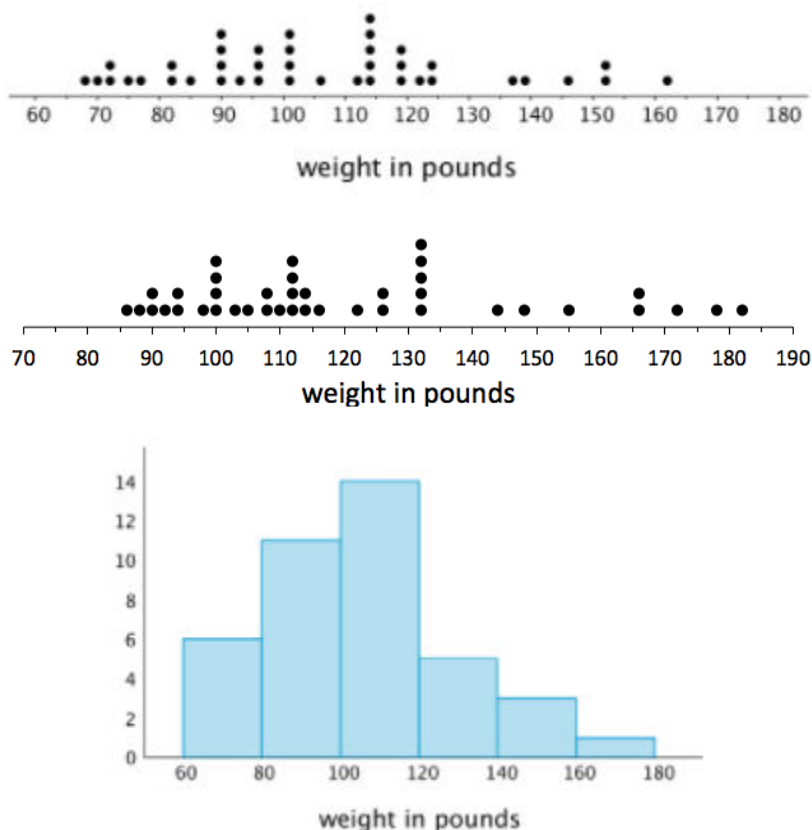
The purpose of this mini-lesson is to review how to read line plots or dot plots from Grade 5, and to introduce histograms as another way of representing similar information. Use this mini-lesson to define the term histogram and to discuss key features that differentiate histograms from line plots or dot plots. In particular, histograms tell you the frequency of data points within a certain range, but they don't tell you exactly what those data points are.

Students learned how to create and interpret line plots in Grade 5, but they may need a reminder. Clarify for scholars that these dot plots are the same as the line plots they saw in Grade 5; they just use different symbols. Use this time to ensure that scholars can accurately read the number of data points, the frequency of each value, and the value of each data point from the line plot before moving on.

Put up the representations and ask scholars which plots could represent the same data sets. Give them time to think independently and discuss with a partner before sharing, and press scholars to explain specific features of the data sets that are or are not the same in the different representations. Once scholars convince one another that the first dot plot matches the histogram, have scholars create their own histogram that matches the second dot plot.

Representations

These data sets correspond to the weights of dogs at different shelters.



Note that in this histogram, each range includes the left-end value but not the right-end value. Tell scholars that this is a decision that mathematicians make, and set the expectation that they clarify this when drawing histograms.

Key Questions

- Could the two dot plots represent the same data set? How can you be sure?
- This other chart is called a histogram. What do you think the numbers along the y-axis of this histogram represent?
- How is this histogram similar to and different from bar graphs that you have seen in the past?
- What do you think are the key features that make a graph a histogram?
- Could this histogram represent the same data set as either of the dot plots? How do you know?
- Is it possible that the histogram represents a different data set from the first dot plot? How could this be?
- How can we create another histogram to represent the data set shown in the second dot plot?

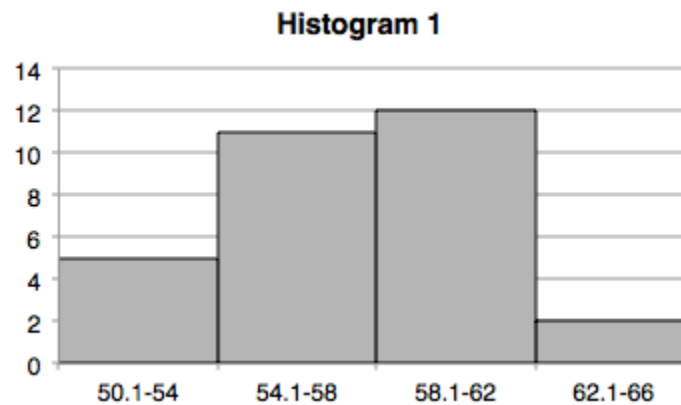
Example 2

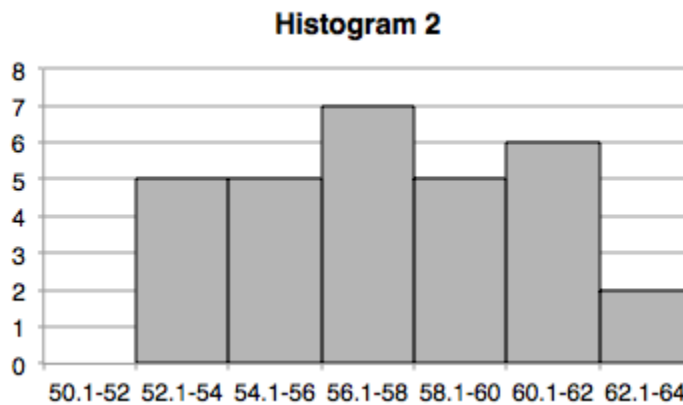
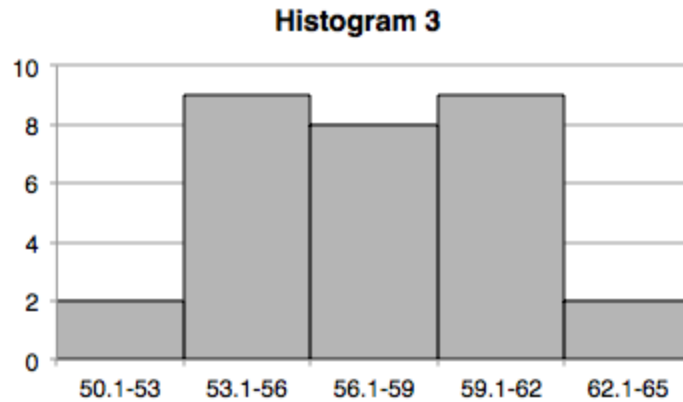
The purpose of this mini-lesson is to have scholars practice reading and comparing histograms. In addition, this mini-lesson is designed to get scholars thinking about what information histograms reveal and what is still left uncertain when data sets are displayed in histograms.

Put up the representations and ask scholars which plots could represent the same data set. Give them time to think independently and discuss with a partner before sharing, and press scholars to explain specific features of the data sets that are or are not the same in the different representations. Press scholars to explain whether the data sets must match, could match, or could not possibly match, and to support their reasoning. Once scholars convince each other that Histograms 1 and 3, but not Histogram 2, could represent the same data set (though we don't know for sure) have them invent their own data set that will match both Histogram 1 and 3.

Representations

These data sets correspond to the average temperatures for each day in a month. Which displays could represent the temperatures for the same month?





Key Questions:

- How is the center and spread of each data set similar or different?
- What is the most frequent value (or bin of values) for each data set?
- Which histograms could represent the same data set? How can you be sure?
- Is it possible for those two histograms to represent different data sets? Why or why not?
- Which histograms could not represent the same data set? How can you prove that?
- How can we design a data set to show that it is possible that Histogram 1 and 3 represent the same month, but not Histogram 2?
- How can you compare histograms that do not use the same intervals?
- How does changing the intervals or the size of the intervals affect the conclusions you draw about a data set?

Example 3

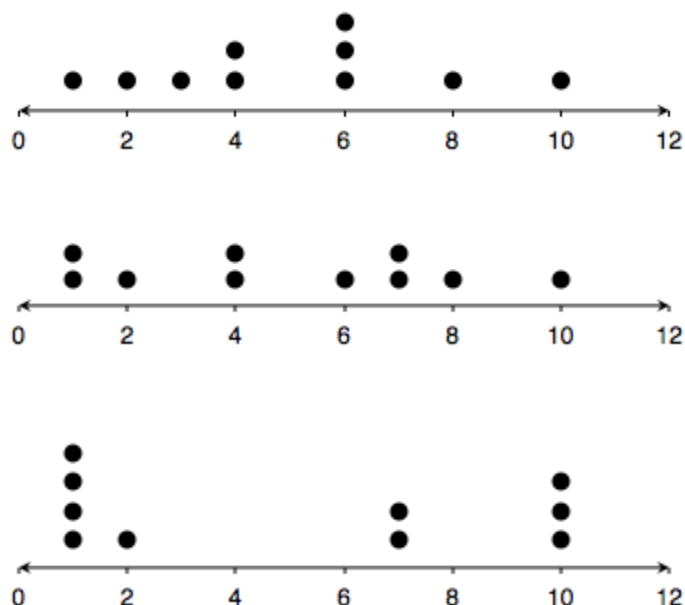
The purpose of this Mini-lesson is to expose scholars to the limits of using mean and range to describe different data sets. This Mini-lesson provides a motivation and justification for exploring the new value you will define with scholars today – the mean absolute deviation (MAD). During this time, your focus should be on solidifying their understanding of mean and range by getting scholars to explain how it's possible for data sets that look so different to have the same mean and range. We do not expect scholars to discover the MAD on their own, but we do want to ensure that they understand why the MAD is important and when it is useful.

Put up the representations and ask scholars which descriptions and displays could match. Give them time to think independently and discuss with a partner before sharing, and press scholars to explain specific features of the data sets that are or are not the same in the different representations. Once scholars convince one another that each dot plot matches the mean and range given, ask them whether

they think those measures are good summaries of these data sets, and what features make the data sets different.

Representations

- A data set has a mean of 5 and a range of 9.



Key Questions

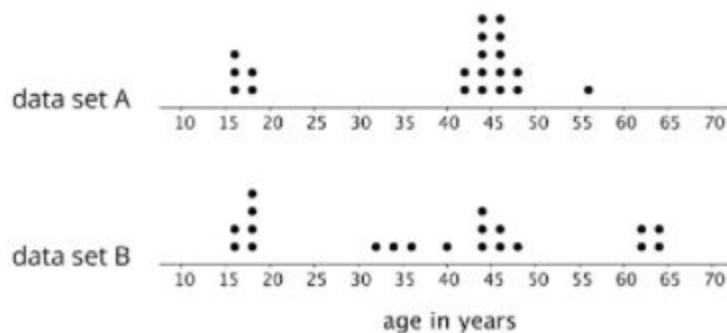
- What is the mean for each data set in the dot plot? How do you know? What does that tell you about the data set?
- For which dot plot do you think the mean is a good representation of a typical value? Explain.
- What is the range for each dot plot? What does that tell you about the data set?
- How is it possible that these data sets have the same range? What information is missing when you know the range of a data set?
- How would you describe the variability or variation in the data for each dot plot?
- These data sets look very different, but when we measure the mean and the range, they appear to be the same. Can you think of a way, besides the range, to measure this variation in order to distinguish between these data sets?

Example 4

The purpose of this lesson is to give scholars practice with estimating mean and MAD visually and interpreting data sets in context. Put up the dot plots and the descriptions and ask scholars which description matches each dot plot. Give scholars time to think independently and discuss with a partner before sharing out. Press them to explain how they can estimate the MAD without calculating, and then have them calculate it to prove their answer.

Once they have paired each dot plot with a situation, have them describe or sketch a data set that would match the remaining description. If time permits, have scholars create a data set with that exact MAD, but if not, have them describe how that data set would compare to the other two.

Representations



- Twenty people - high school students, parents, guardians, and teachers - attended a rehearsal for a high school musical. The mean age was 38.5 years and the MAD was 16.5 years.
- Family members of the players usually come to watch the high school soccer team practice. One evening, twenty people watched the practice. The mean age was 38.5 years and the MAD was 12.7 years.
- Twenty people watch a chess tournament. The mean age was 38.5 years, but the MAD was 20 years.

Key Questions:

- What does MAD stand for? What does it represent or tell us about a data set?
- How can you compare the MAD of these data sets without calculating?
- How can you prove that this description matches this data set?
- Do data sets with larger ranges always have larger MADs too? Explain your reasoning.
- Which description does not match either data set? How do you know?
- How would that data set compare to the two that we have here? How could you create your own data set to match that description?

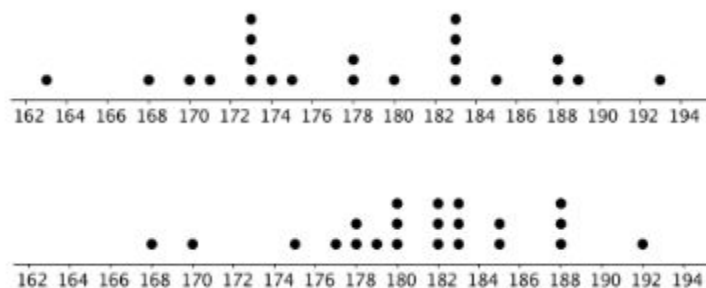
Example 5

The goal of this mini-lesson is for scholars to practice finding the median and using the mean and the median to describe a “typical” value of a data set.

Put up the representations and medians and ask scholars which ones match. Give scholars time to think independently then discuss with a partner before sharing out.

Representations

These dot plots represent the heights, in centimeters, of the first 22 presidents and the next 22 presidents.



- Median = 178 centimeters
- Median = 182 centimeters

Key Questions:

- Without calculating, which dot plot do you think has the greater median?

- How can you find the median of these dot plots?
- How would the median of each data set change if you remove the minimum value?
- What is the mean for each data set, and how does it compare to the median?
- Which measure do you think is a better representation of a “typical” value for each data set? Why?

Example 6

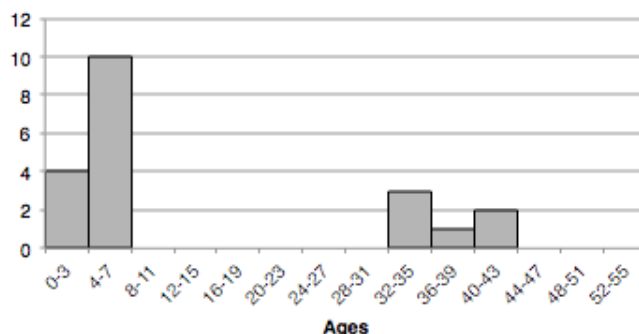
The purpose of this Mini-lesson is to have scholars practice interpreting histograms and finding median and interquartile ranges (IQR). Focus your discussion on what scholars can infer from the histograms, and what they still don’t know about these data sets.

Show scholars one of the histograms first and ask whether it is possible to calculate the median exactly from this representation, and if not, how scholars can use this representation to estimate the median. Once scholars have reasoned about how to use the frequencies to estimate the median, show all the representations and ask them which description matches which histogram. Give them time to think independently and discuss with a partner before sharing, and press scholars to explain how they can estimate the IQR for each histogram.

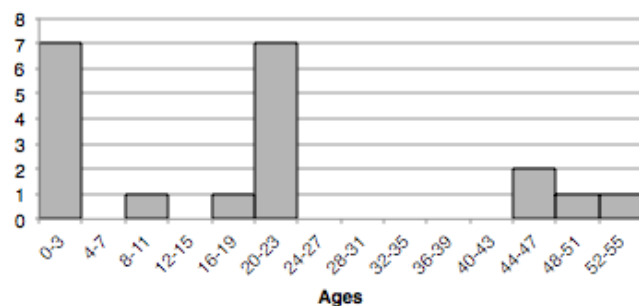
Representations

Tell scholars that the histograms below show the ages of people at a birthday party and at a park.

Ages of People at Birthday Party



Ages of People at Park



- Median age = 20 years and IQR = 22 years
- Median age = 5 years and IQR = 30 years

Key Questions:

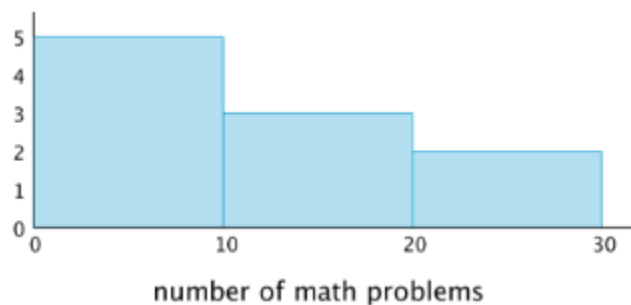
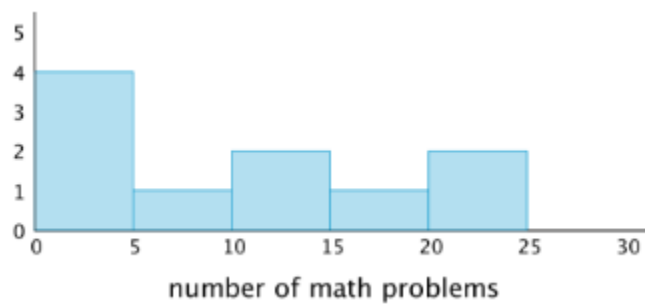
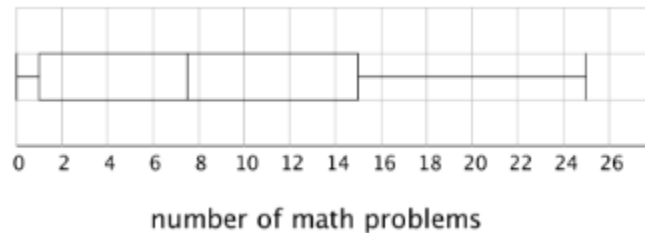
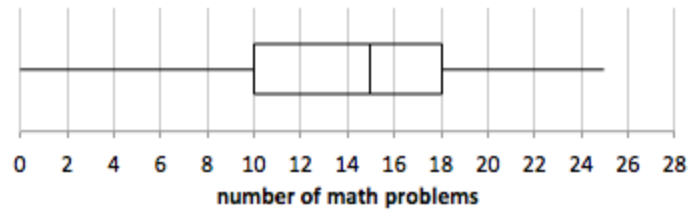
- Can you find the exact IQR from a histogram? Why or why not?
- How can you estimate the minimum possible IQR from a histogram?
- How can you estimate the maximum possible IQR from a histogram?
- Do you think it is easier to estimate the mean or median using histograms? Explain your reasoning.
- Do you think it is easier to estimate the MAD or the IQR using histograms? Explain your reasoning.

Example 7

The purpose of this mini-lesson is to have scholars practice comparing data sets that are represented in different ways and making inferences about these data sets. Put up the representations and ask scholars which ones could represent the same data set. They should quickly notice that the two histograms could represent the same data set, so press them to explain whether the box plots could also represent these data sets and to use specific features of the representations to justify their answers.

Representations

Each plot represents the number of math homework problems scholars completed over ten days.



Key Questions:

- Is it impossible for any of these representations to match the same data sets? Which ones?
- Is it guaranteed that any of these representations match the same data sets? Why or why not?
- How would you describe the symmetry and spread of these data sets?
- For these data sets, how many data points are in each section of the box plot? How can that help you compare them to the histograms?
- What can you infer about data sets that are represented in box plots? Histograms?
- Can you estimate the mean of these data sets? Which representation is the easiest to use?

Grade 7

Example 1

The purpose of this routine is to give students practice with pinpointing what must match up between complicated situations and the simulations that could be used to analyze them.

Display all situations and simulations, and ask which simulation could represent each situation. Press scholars to use theoretical and experimental probabilities to clearly explain why the simulations match. For the situation that doesn't match either simulation, have scholars design their own simulations and share out.

Situations	Simulations
• The chance that any child born is a girl is $\frac{1}{2}$. What is the probability that a person who has 5 grandchildren has no more than 1 granddaughter? • An automobile factory noticed a flaw. They estimate that approximately 10% of cars assembled over the last week have defective brakes. Five cars from that factory are shipped to a dealership near your school. What is the probability that no more than one of these cars have defective brakes? • The staff at a local park estimate that there is a 40% chance of seeing an eagle on any given day that you visit the park. If your class plans to visit the park on 5 different days, what is the probability that you will see an eagle on 1 or more days?	• In a bag, there are 9 blue marbles and 1 red marble. Choose a marble at random, record the color, and replace it. Do this 5 times to record 5 colors, then repeat this process many times. Count the number of times you choose 1 red marble or no red marbles out of 5. • Make a spinner with 6 blue sections and 4 green sections, all the same size. Spin the spinner 5 times and record the color each time, and repeat this process. Count the number of times you get 1 or more greens.

Key Questions:

- What are the possible outcomes of this situation? Are they equally likely?
- What are the possible outcomes in this simulation? Are they equally likely?
- What is similar and different about these two simulations?
- Which outcome or outcomes in the simulation match each outcome in the realistic situation? How do you know?
- How can we design an experiment to match the other realistic situation?
- What would you need to do to find the theoretical probability of each situation?

Example 2

The purpose of this routine is for scholars to practice explaining how random events can be used to simulate complicated situations. Build on your conversations from Lesson 3 by getting scholars to think about whether they would have an easier time envisioning the possible outcomes and relative likelihood of each for the situations or the simulations, and about how envisioning all possible outcomes changes when the event is repeated. These ideas will be solidified during the Discourse.

Display all situations and simulations, and ask which simulation could represent each situation. For the situation that doesn't match either simulation, have scholars design their own simulations and share out.

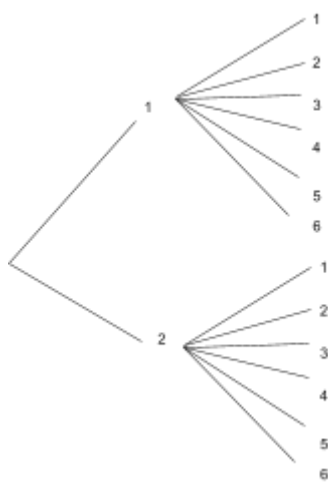
Situations	Simulations
• In a small lake, 25% of the fish are female. When you catch and release 4 fish, what is the probability that exactly 3 fish are female? • Elena plays basketball and she makes 80% of the free throws she takes. What is the probability that Elena will make exactly 3 out of her next 4 free throws? • Chelsea and Daniella sing in the school choir. Before each concert, there is an equal chance that the choir teacher will select Chelsea or Daniella to sing the solo. What is the probability that Chelsea will be selected to sing the solo at exactly 3 out of 4 concerts?	• Fill a bag with two red marbles and two blue marbles. Without looking, choose a marble and replace it four times in a row, and record the color each time. Repeat this process many times, and find the fraction of trials in which exactly 3 blue marbles appear. • Make a spinner with four equal-sized sections labeled 1, 2, 3, and 4. Spin the spinner 4 times and record the outcomes. Repeat this process many times, and find the fraction of the trials in which a 4 appears exactly 3 times.

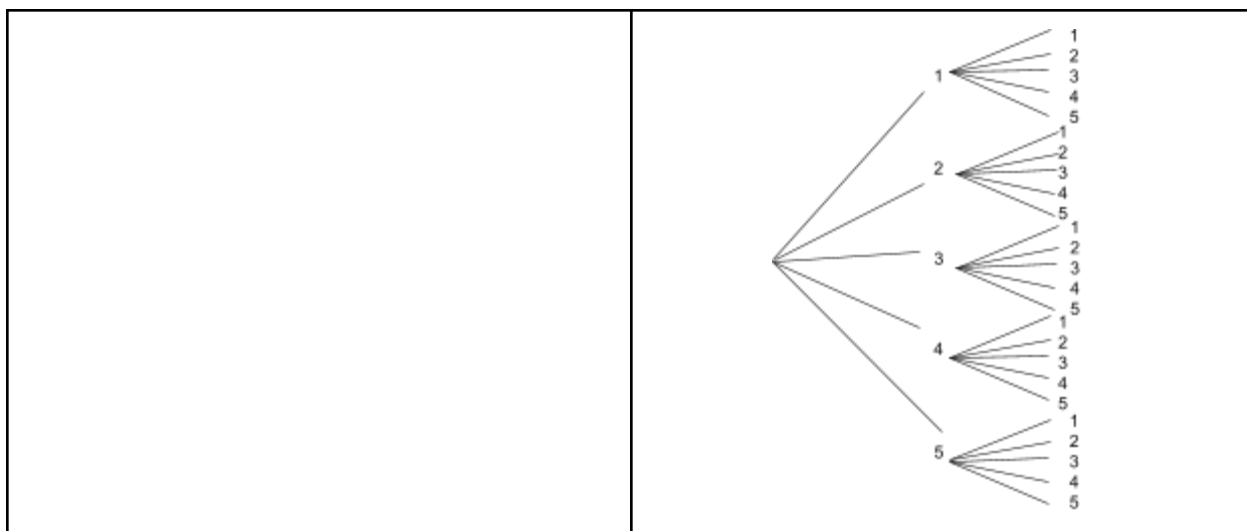
Key Questions:

- What are the possible outcomes of this situation? Are they equally likely?
- What are the possible outcomes in this simulation? Are they equally likely?
- Is it easier for you to envision the possible outcomes and the likelihood of each one for the simulations or the situations? Why?
- Which outcome or outcomes in the simulation match each outcome in the realistic situation? How do you know?
- How can we design an experiment to match the other realistic situation?

Example 3

The purpose of this Mini-lesson is for scholars to practice visualizing sample spaces of compound events and interpreting tree diagrams to determine which ones match. Once scholars agree on which event and sample space match below, have them represent the sample space of the remaining event and describe an event that matches the remaining tree diagram.

Events	Sample Spaces
• Spin 2 spinners with equal-sized sections labeled 1 through 6. • Spin 2 spinners with equal-sized sections labeled 1 through 5.	 <pre> graph LR A1[1] --- B1[1] A1 --- B2[2] A1 --- B3[3] A1 --- B4[4] A1 --- B5[5] A1 --- B6[6] A2[2] --- B7[1] A2 --- B8[2] A2 --- B9[3] A2 --- B10[4] A2 --- B11[5] A2 --- B12[6] </pre>



Key Questions

- What does each number in this tree diagram represent?
- What outcome does this branch of the tree diagram represent?
- Does the outcome of the first event affect the outcomes of the second event in these cases? How do you know?
- How can you prove whether or not this tree diagram includes all possible outcomes, with none repeated and none missing?

Example 4

Scholars should have calculators during this time.

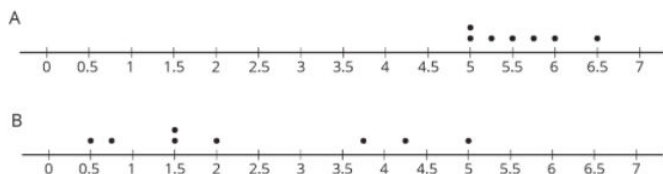
The purpose of this Mini-lesson is to review how to calculate mean and mean absolute deviation (MAD). Open by asking scholars what they remember about mean and mean absolute deviation from Grade 6. At this point, they should say things like, “Mean is the average” and “MAD tells us something about how spread apart the points are.” Put up all five representations and ask scholars to match the numerical values to the corresponding dot plots.

Scholars learned how to calculate these values in Grade 6, but they will likely need a reminder. Push scholars to interpret these values informally (i.e., a larger MAD means more variation) before reviewing how to calculate the MAD of the first two data sets (dot plots A and B). Use these calculations to have scholars evaluate their interpretations, and finish the Mini-lesson by having scholars calculate the mean and MAD of the third dot plot (dot plot C) on their own.

Representations

Dot Plots

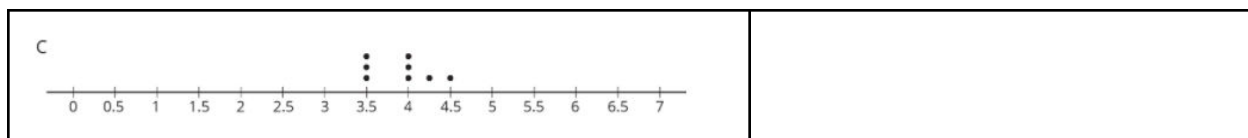
These data sets correspond to the length, in minutes, of songs on different CDs.



Mean and MAD

Mean: 5.57 minutes and MAD: 0.44 minutes

Mean: 2.41 minutes and MAD: 1.44 minutes



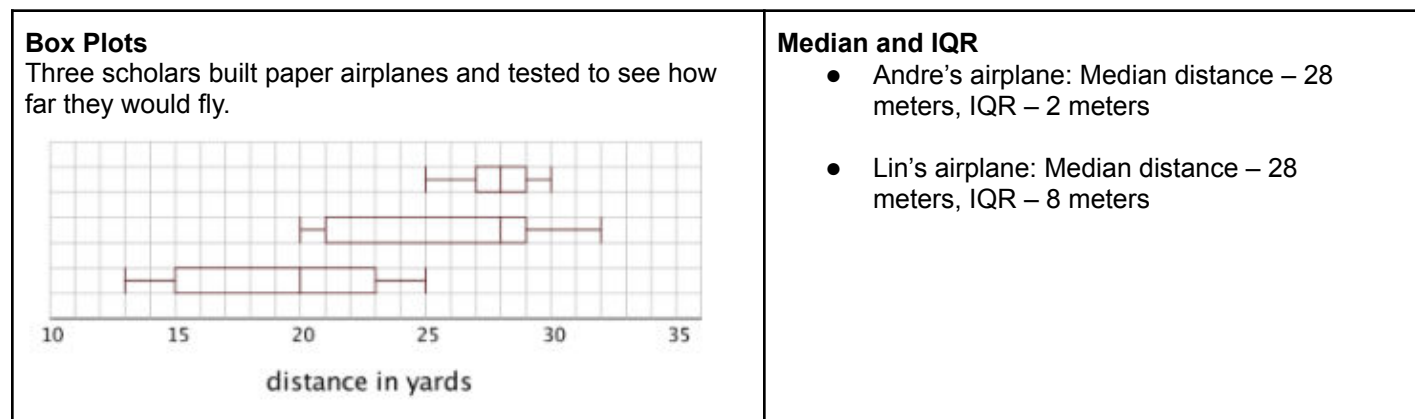
Key Questions

- Describe what one individual dot represents in this context.
- How can you estimate the mean visually without calculating?
- What does the MAD tell us about a data set? How can you tell which data sets have a larger MAD without calculating?
- What conclusions can you draw from the dot plots? What conclusions can you draw when you know the exact mean and MAD?
- How can you calculate the mean and MAD from a data set or dot plot?

Example 5

The purpose of this Mini-lesson is to review the meaning of median and interquartile range (IQR), and how those values are represented in box plots. First, ask scholars what they remember about what the median and interquartile range tell you about a data set. Then show all the representations below and ask them which box plots match which person's data. Finish the Mini-lesson by telling scholars that the third box plot represents Maria's paper airplane and asking them to find the median and IQR from the box plot.

During this discussion, label the median and IQR directly on each box plot and leave this up for your class to reference during the Explore. They will return to analyzing and comparing box plots later on in this lesson.



Key Questions:

- How is the median similar to and different from the mean?
 - Make sure to focus this conversation not just around the differences in how these values are calculated, but what those differences mean for how these values are interpreted as well.
- How is the IQR similar to and different from the MAD? How is it similar to and different from the range?
- Where do you see the median and IQR in each box plot?
- What fraction of trials was above the median? What fraction was below?
- What does the line on the far left represent in each box plot? What does that represent in this context?
- What does the line on the far right represent in each box plot?

- What does the box in the middle represent in each box plot? What fraction of trials fit inside that box?

Grade 8

Example 1

The goal of this mini-lesson is to introduce scholars to factored form and to have them articulate how and why the factors of quadratic functions are related to the x-intercepts. Throughout this lesson, you must stay focused on interpreting factors and x-intercepts in terms of multiplication. While some scholars might recognize that the zeros of the function are equivalent to p and q in the factored form $y = (x - p)(x - q)$, it is much more important that scholars are able to use an understanding of the meaning of x-intercepts and the zero property of multiplication to prove their thinking. Because of the extensive precision errors scholars make when subtracting negative numbers, simply memorizing how to use the values of p and q to graph a quadratic function is inevitably going to lead to mistakes.

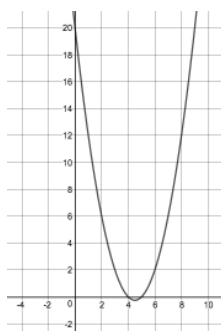
Launch the mini-lesson by showing scholars the first equation $y = (x + 4)(x + 5)$ and asking them what type of function they think this will be. Ask them to think about when they have seen similar expressions before, and remind them that they should use the area model to keep track of their work when expanding the expression to rewrite this function.

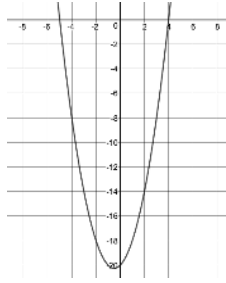
Once scholars have re-written this function in standard form, ask them what they know about the graph of this function. Give them time to think independently then discuss with a partner before sharing. As they share out, press them to identify specific key features of this graph (the shape, y-intercept, direction of the parabola, etc) and record their observations. Use this Launch to introduce the term factored form to describe the way that this equation is written as a product of two factors. Make sure that scholars understand that a “factor” in this context means the same thing as the “factors” they are used to finding for whole numbers.

Then put up the additional equations and graphs and ask scholars which equations and graphs represent the same functions. Scholars should have an opportunity to consider the representations individually and then in partners before discussing as a whole class. As scholars share out, ensure that they use precise academic language and prove their thinking.

Representations

- $f(x) = (x + 4)(x + 5)$
- $f(x) = (x + 8)(x + 5)$
- $f(x) = (x - 4)(x + 5)$





Key Questions

- How can we prove which equation matches which graph?
- How can we use our understanding of y-intercepts to prove our thinking? How about a graphing calculator?
 - Begin your discussion by focusing on scholars who expanded the equations and identified the y-intercepts as a key way to determine which graph matches which equation. Once scholars have convinced each other that the first two equations match the graphs, discuss how the third equation is similar and different to the first and ask scholars to predict in what ways the graph will be similar and different to the graph of the first equation.
 - Then give scholars time to graph this equation in their graphing calculators to test their predictions before asking them to compare and interpret the x-intercepts of these graphs.
- How do the x-intercepts of these graphs compare?
- What do the x-intercepts of each graph represent?
- What would need to be true about the factors in order for the equation to equal 0? Why?
- How are the factors and factored form of these equations related to the x-intercepts of these graphs?
- When graphing a quadratic function would you prefer for the function to be written in standard form or factored form? Why?

Example 2

The purpose of this mini-lesson is to consider what the graph of a quadratic function that is a perfect square when factored looks like and why. Scholars should leave the Mini-lesson understanding that this is a unique type of quadratic function that only touches the x-axis, but doesn't cross it. Because both factors are the same, there is only one value that makes the output zero. Therefore this function has just one x-intercept, which is also the vertex.

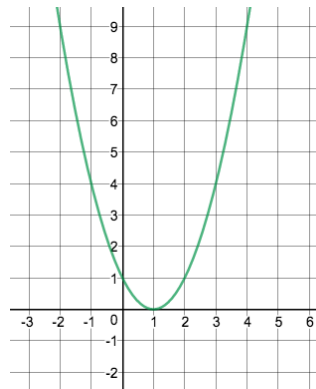
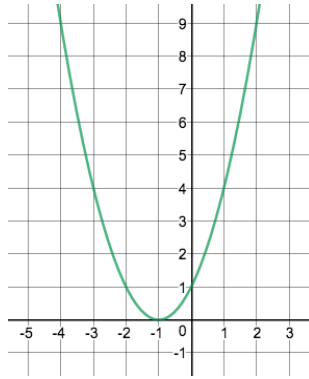
Put up all of the representations. The goal is for students to determine which representations are connected (i.e., which equations match which graphs). Scholars should have an opportunity to consider the representations individually and then in partners before discussing as a whole class. To review what scholars learned in Lesson 3, you may begin by showing just the factored form of the first equation $f(x) = (x - 1)(x - 1)$ and asking scholars what predictions they can make about the graph based on that equation before revealing the other representations.

Representations

$$f(x) = (x - 1)^2$$

$$f(x) = x^2 + 6x + 9$$

$$f(x) = x^2 + 2x + 1$$



Key Questions

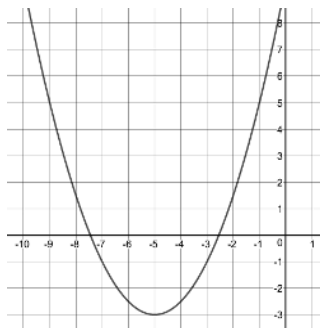
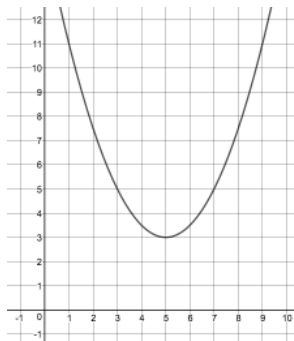
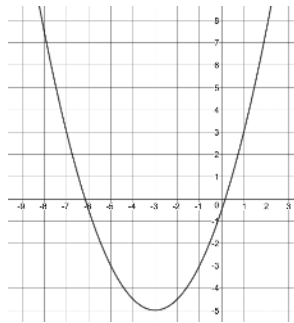
- How do you know these two representations match?
- How can we use our understanding of factoring and x -intercepts to prove our thinking? How about a graphing calculator?
 - As scholars discuss the fact that there is only one x -intercept, be sure to define the term vertex as the minimum or maximum point of a parabola. Ask scholars how they can use either the standard or factored form of the function to decide whether the vertex will be a minimum or maximum.
- How are these functions similar to and different from the functions we explored yesterday?
- What would the parabola of the final function look like? How do you know?

Example 3

The purpose of this mini-lesson is to reinforce scholars' understanding of the relationship between a graph of a quadratic function and the algebraic representation of the same function in vertex form.

Put up all of the representations. The goal is for students to determine which representations are connected (i.e., which equations match which graphs). Scholars should have an opportunity to consider the representations individually and then in partners before discussing as a whole class.

Representations



$$f(x) = \frac{1}{2}(x + 5)^2 - 3$$

$$f(x) = \frac{1}{2}(x + 3)^2 - 5$$

Key Questions

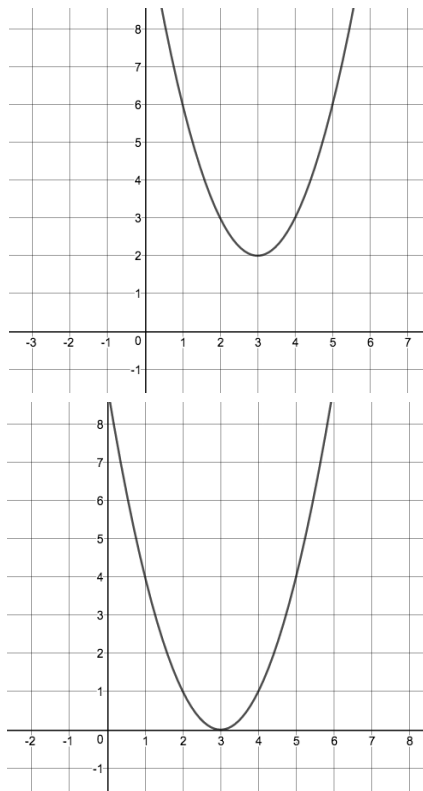
- How do you know these two representations match?
- Without trying to memorize which coordinate of the vertex is h and which is k in vertex form, how can we prove our thinking? How about with a graphing calculator?
- How can you prove that the leading coefficient (the $\frac{1}{2}$) in this equation is accurate?
- What might the equation for the final function be in vertex form?
- Do we have enough information to write this function exactly?
- What did you learn about quadratic functions from this Mini-lesson?

Example 4

Scholars should walk away from this Mini-lesson recognizing that not all quadratic functions are factorable. Graphically, scholars should recognize that a quadratic function is not factorable if the function does not cross the x -axis and therefore does not have any (real) zeros. Algebraically, scholars should understand that if there are no terms that add together to get the b term of the standard equation and simultaneously multiply together to get the c term of the standard equation the function is not factorable. Through this Mini-lesson scholars should recognize that one of the benefits of vertex form is that, like standard form, it can be used to algebraically represent any quadratic function.

Put up all of the representations. The goal is for students to determine which representations are connected (i.e., which equations match which graphs). Scholars should have an opportunity to consider the representations individually and then in partners before discussing as a whole class.

Representations



$$y = (x - 3)^2 - 1$$

$$y = (x - 3)^2$$

$$y = (x - 3)^2 + 2$$

Key Questions

- How do you know these two representations match?
- How can we use our understanding of vertex form to prove our thinking? How about a graphing calculator?
- What would the graph of the final equation look like? How do you know?

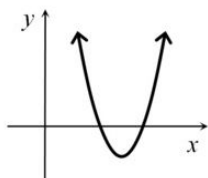
- Could all of these functions be written in factored form? Why or why not?
- Could all of these functions be written in standard form? Why or why not?
- What would the parabola of the final function look like? How do you know?
- What did you learn about quadratic functions from this Mini-lesson?

Example 5

Put up all of the representations. The goal is for students to determine which representations are connected (i.e., which equations match which graphs). Scholars should have an opportunity to consider the representations individually and then in partners before discussing as a whole class.

This version of Connecting Representations is different from what scholars are used to. Instead of having approximately the same number of graphs and equations, this time they are given one graph with no scale and asked to determine which equations could match. The purpose of this mini-lesson is for scholars to practice using different forms of quadratic functions to identify key features of a graph as well as to build reasoning skills.

Representations



$f_1(x) = (x + 12)^2 + 4$	$f_5(x) = -4(x + 2)(x + 3)$
$f_2(x) = -(x - 2)^2 - 1$	$f_6(x) = (x + 4)(x - 6)$
$f_3(x) = (x + 18)^2 - 40$	$f_7(x) = (x - 12)(-x + 18)$
$f_4(x) = (x - 12)^2 - 9$	$f_8(x) = (24 - x)(40 - x)$

Key Questions

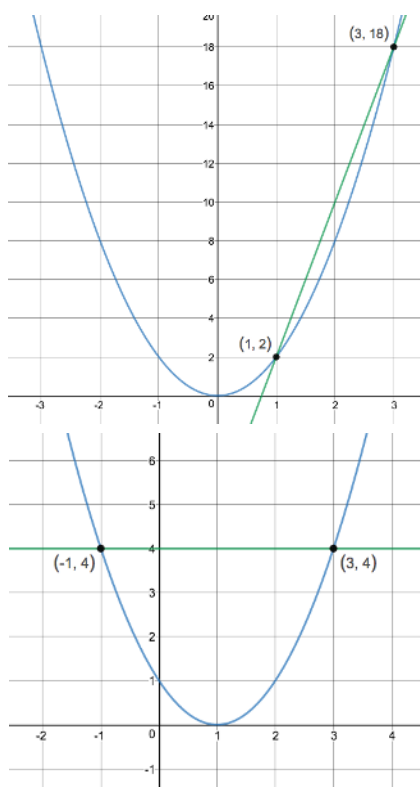
- Which equations could match this graph? How do you know?
- Are we going to be able to prove that these equations match this graph exactly? Why or why not?
- Are there any equations that you can immediately rule out as not matching the graph? Why?
- What information can you gain from looking at a quadratic function written in factored form compared to a quadratic function in vertex form?
- Is one form more helpful than the other for this task? Why or why not?

Example 6

The purpose of this mini-lesson is to strengthen scholars' understanding that any equation can be solved by graphing each side and finding where lines intersect. Scholars should use what they learned in the previous lesson to choose an efficient algebraic strategy to prove their thinking.

Put up all of the representations. The goal is for students to determine which representations are connected (i.e., which equations match which graphs). Scholars should have an opportunity to consider the representations individually and then in partners before discussing as a whole class.

Representations



$$\begin{aligned}(x - 1)^2 &= 4 \\ 3 &= x^2 + 2x \\ 2x^2 &= 8x - 6\end{aligned}$$

Key Questions

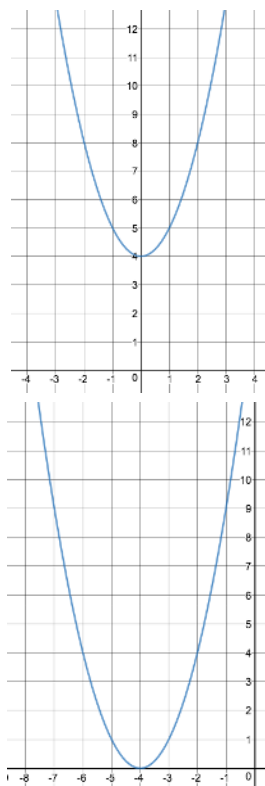
- How do you know these two representations match?
- How can we prove our thinking algebraically?
- What did you learn about solving quadratic equations from this Mini-lesson?
- How could we graphically find the solution to the final function?

Example 7

The purpose of this Mini-lesson is to introduce scholars to the word transformation to describe how a function has changed from its original form. Scholars should be able to use graphs and tables to identify the how the function $y = x^2$ was transformed. Scholars will recognize that going from x^2 to $x^2 + 4$ shifts the function up and going from x^2 to $(x + 4)^2$ shifts the function to the left.

Put up all of the representations. The goal is for students to determine which representations are connected (i.e., which equations match which graphs). Scholars should have an opportunity to consider the representations individually and then in partners before discussing as a whole class.

Representations



$$y = x^2$$

$$y = x^2 + 4$$

$$y = (x + 4)^2$$

Key Questions

- How do you know these two representations match?
- How can we use a table to prove our thinking? How about a graphing calculator?
- How are these representations similar? How are they different?
- If we think of $y = x^2$ as our original function, how would you describe how the function has been changed or transformed with each new equation?